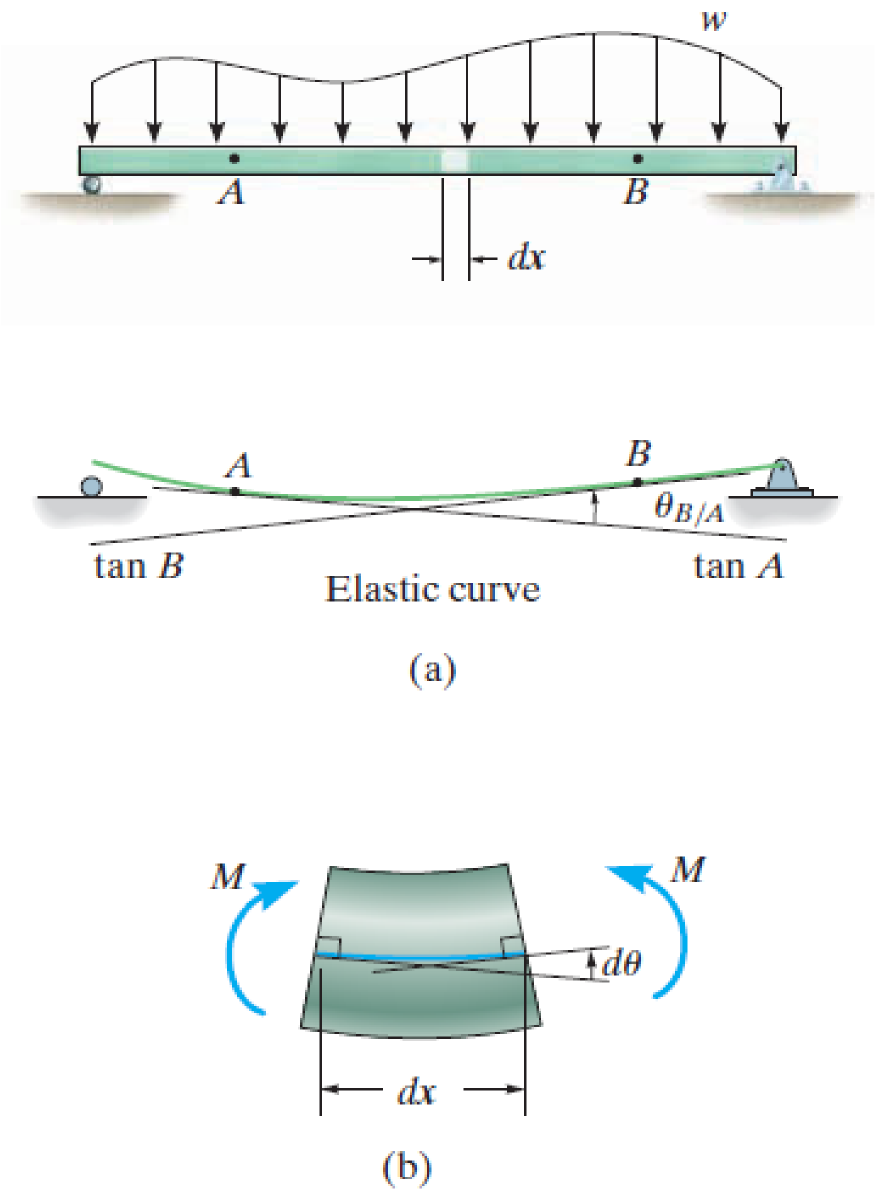
Slope and Displacement by the Moment Area Method

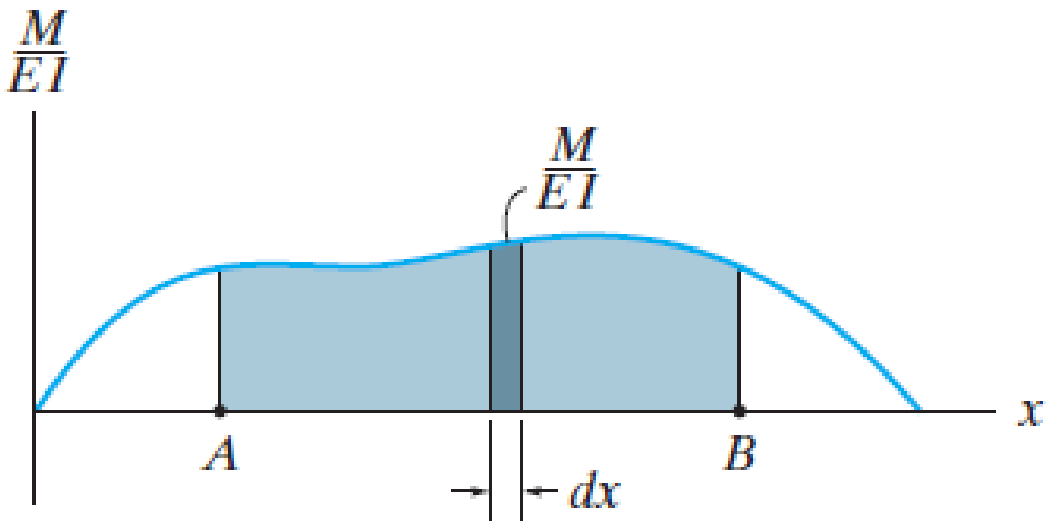
The moment-area method provides a semigraphical technique for finding the slope and displacement at specific points on the elastic curve of a beam or shaft. Application of the method requires calculating areas associated with the beam's moment diagram; and so if this diagram consists of simple shapes, the method is very convenient to use. Normally this is the case when the beam is loaded with concentrated forces and couple moments



$$EI\frac{d^2v}{dx^2} = EI\frac{d}{dx}\left(\frac{dv}{dx}\right) = M$$

Since the *slope* is *small*, $\theta = dv/dx$, and therefore

$$d\theta = \frac{M}{EI}dx$$

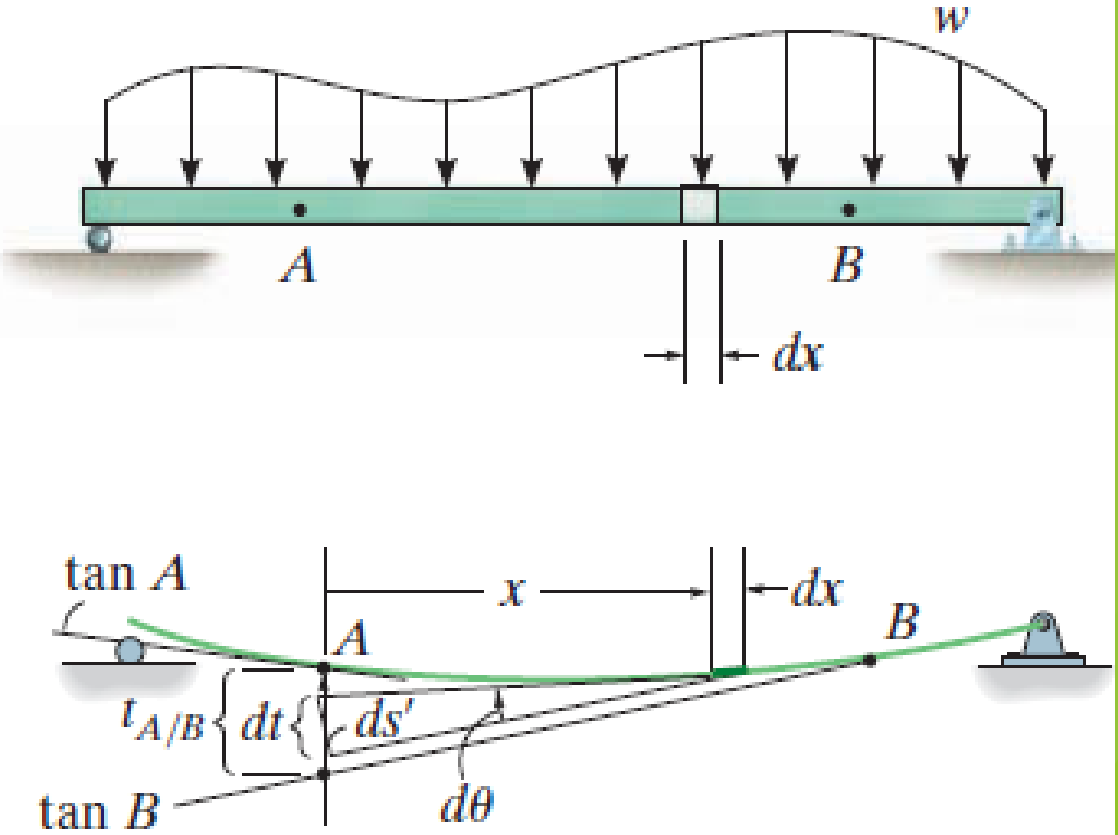


$\frac{M}{EI}$ Diagram

(c)

$$\theta_{B/A} = \int_A^B \frac{M}{EI}dx$$

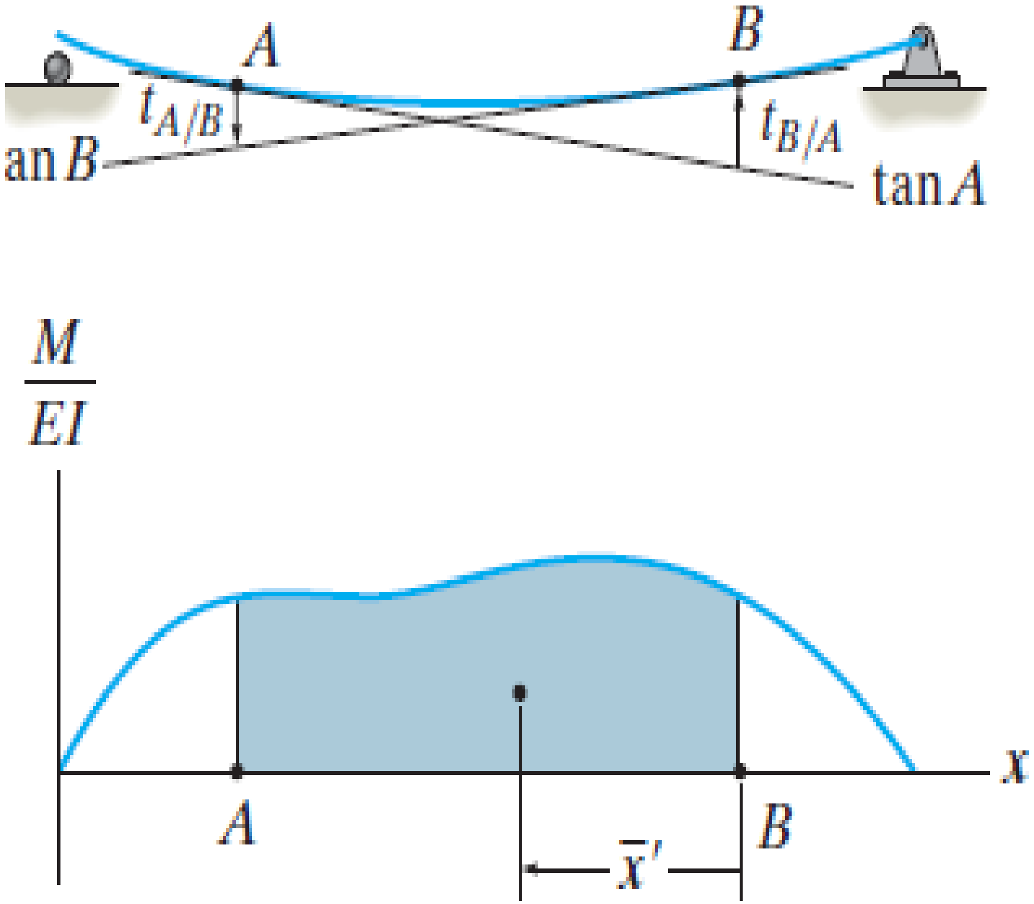
Theorem 1: The angle between the tangents at any two points on the elastic curve equals the area under the MEI diagram between these two points. The notation $\theta_{B/A}$ is referred to as the angle of the tangent at B measured with respect to the tangent at A. From the proof it should be evident that this angle is measured counterclockwise, from tangent A to tangent B, if the area under the M/EI diagram is positive. Conversely, if the area is negative, or lies below the x axis, the angle $\theta_{B/A}$ is measured clockwise from tangent A to tangent B.



(a)

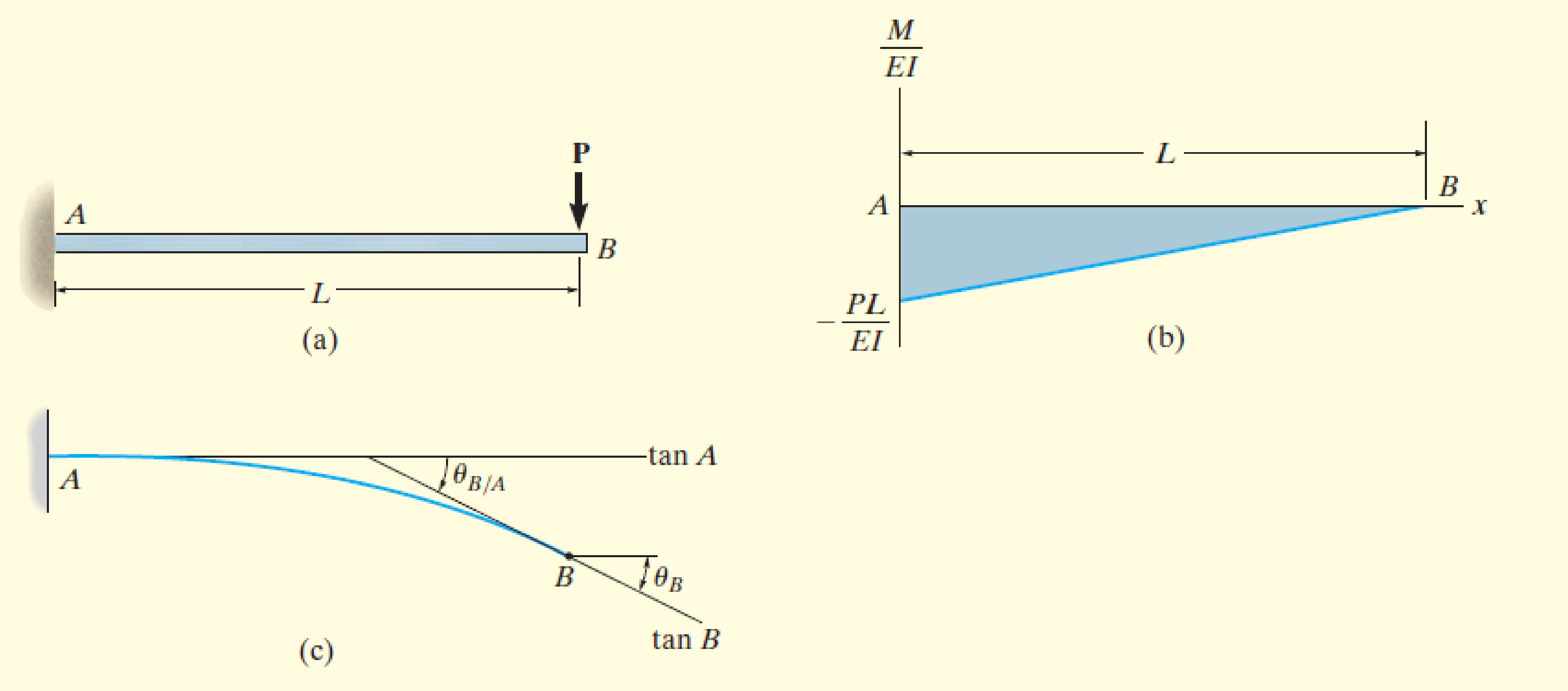
$$t_{A/B} = \int_A^B x \frac{M}{EI} dx$$

Theorem 2: *The vertical distance between the tangent at a point (A) on the elastic curve and the tangent extended from another point (B) equals the moment of the area under the M/EI diagram between these two points (A and B). This moment is calculated about the point (A) where the vertical distance* $t_{A/B}$ *is to be determined.*



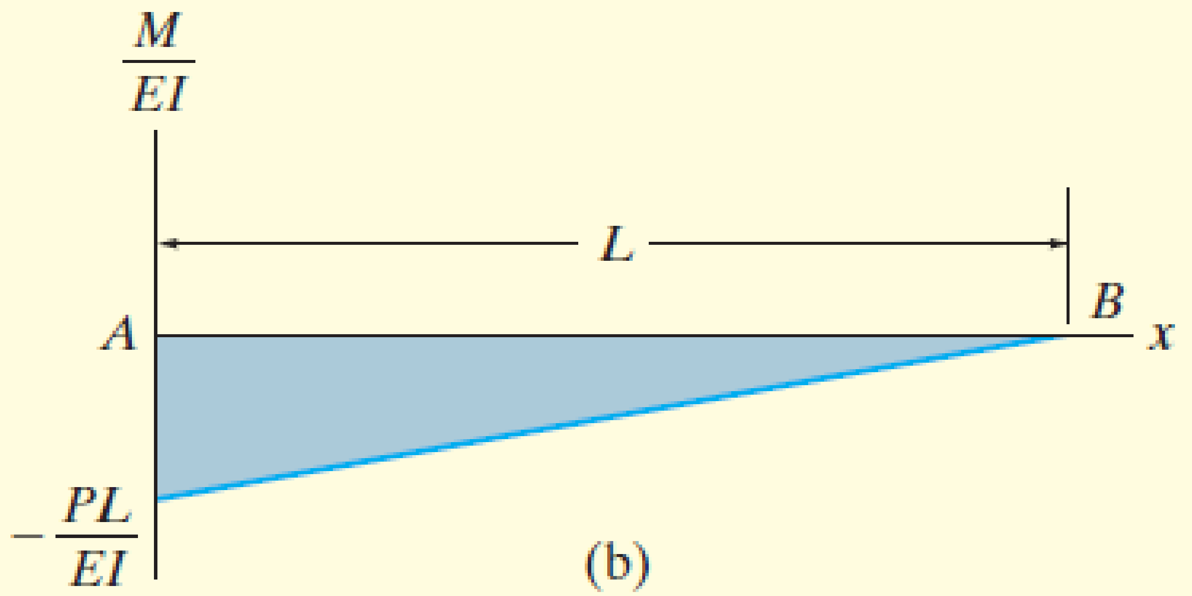
(c)

Determine the slope of the beam shown in Figure below at point *B*. *EI* is constant.



Solution:-

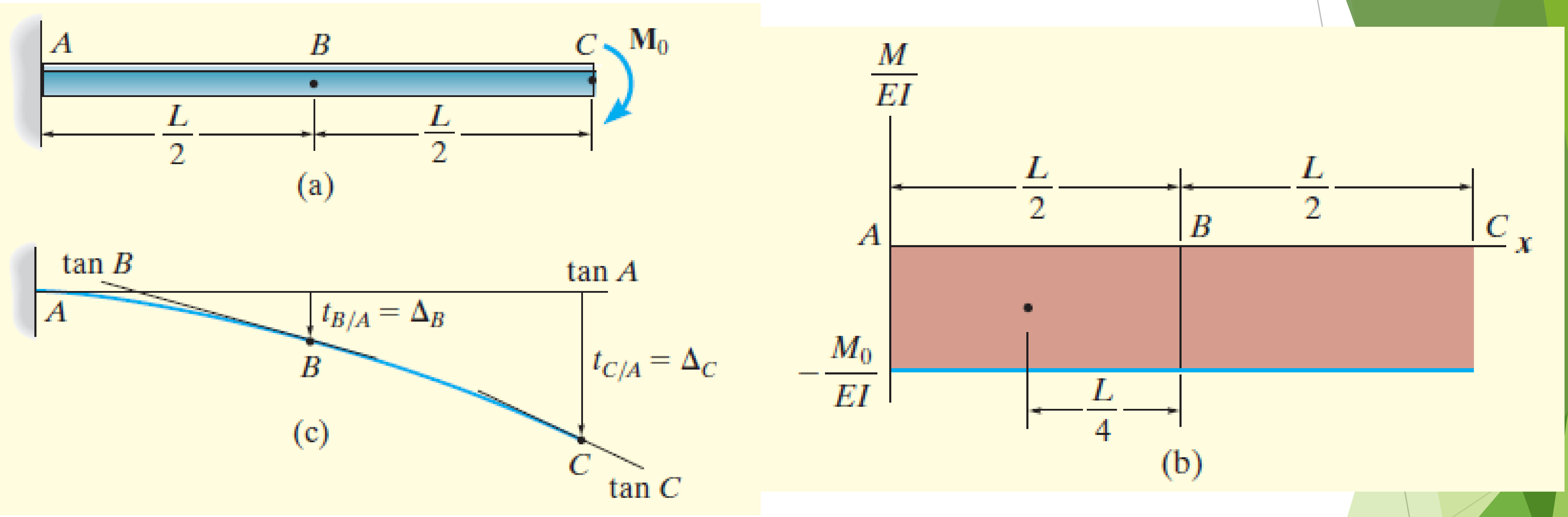
$$\theta_B = \theta_{B/A}$$



$$\theta_B = \theta_{B/A} = \frac{1}{2}\left(-\frac{PL}{EI}\right)L$$

$$= -\frac{PL^2}{2EI}$$

Determine the displacement of points *B* and *C* of the beam shown in Figure below. *EI* is constant.



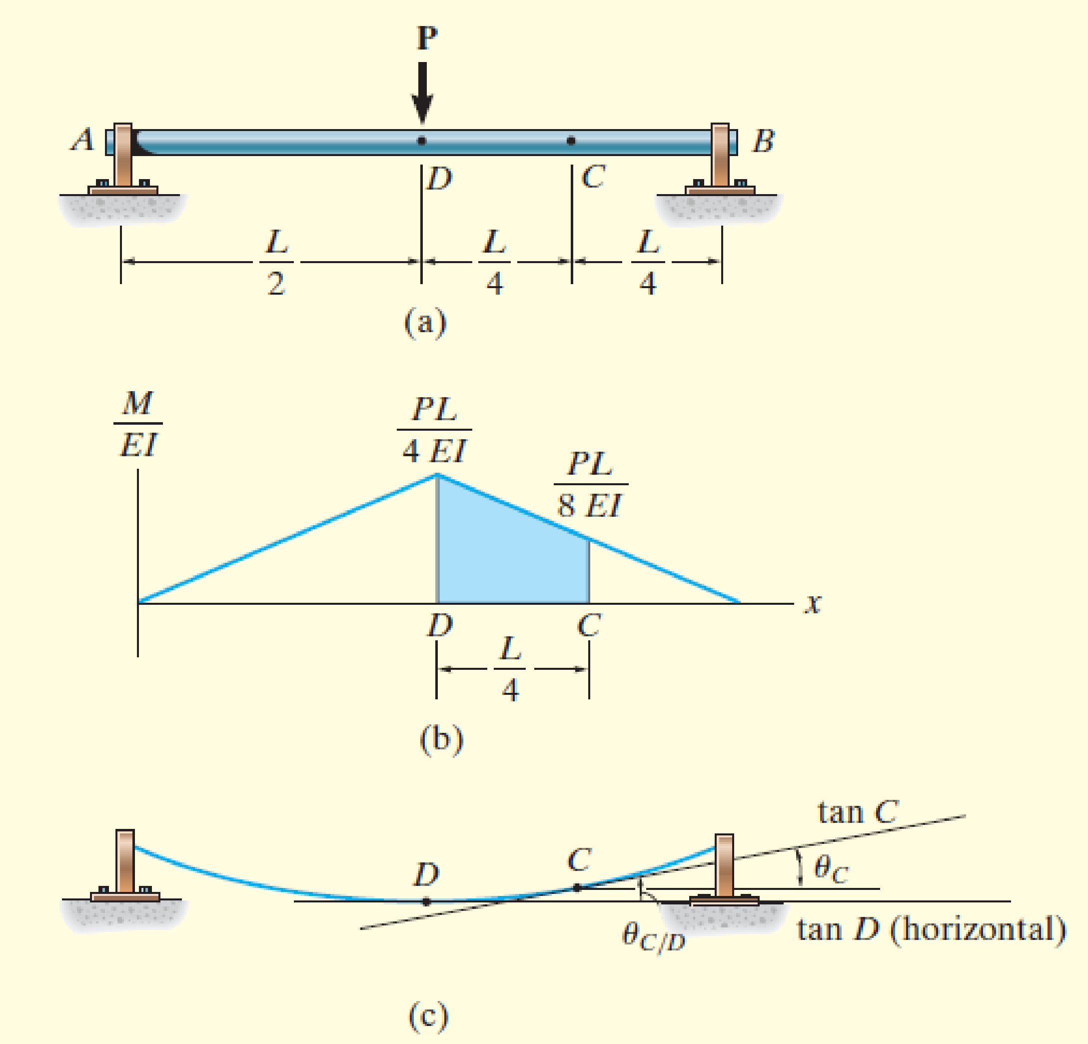
$$\Delta_B = t_{B/A}$$

$$\Delta_C = t_{C/A}$$

$$\Delta_B = t_{B/A} = \left(\frac{L}{4}\right)\left[\left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right)\right] = -\frac{M_0L^2}{8EI}$$

$$\Delta_C = t_{C/A} = \left(\frac{L}{2}\right)\left[\left(-\frac{M_0}{EI}\right)(L)\right] = -\frac{M_0L^2}{2EI}$$

Determine the slope at point C of the shaft in Figure below. EI is constant.



$$\theta_C = \theta_{C/D}$$

$$\theta_C = \theta_{C/D} = \left(\frac{PL}{8EI}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{PL}{4EI} - \frac{PL}{8EI}\right)\left(\frac{L}{4}\right) = \frac{3PL^2}{64EI}$$